Homework 3. Linear Algebra. Spring 2022. Prof. Pineiro

Print Name: ______________________________

1. Given the matrix: $A = \begin{pmatrix} 2 & 1 \\ -1 & 0 \\ -2 & 3 \end{pmatrix}$
   (a) Compute AA^t.
   (b) Compute A^tA.
   (c) Show that A^tA is invertible and find its inverse.
   (d) Show that AA^t is not invertible.

2. Determine whether or not the vectors $\vec{v}_1 = \begin{pmatrix} 4 \\ -3 \\ 3 \end{pmatrix}$, $\vec{v}_2 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ and $\vec{v}_3 = \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}$ are linearly independent.

3. Find the equation in $\mathbb{R}^4$ of the vector space W generated by the vectors

$$v_1 = (-2, 2, 1, 3) \quad v_2 = (1, 1, 2, 0) \quad v_3 = (0, 3, 3, 3).$$

   Determine the dimension of W.

4. Suppose that the system $\{v_1, v_2, v_3\}$ is a basis of a vector space V. Determine whether or not the following systems also represent basis of V. Prove your answer or find counterexample!
   (a) $\{v_1 + v_2 - 2v_3, v_1 - v_3, v_2\}$
   (b) $\{v_1, v_1 + v_2 + v_3\}$
   (c) $\{v_1, v_1 + v_2 + v_3, v_2 + v_3\}$

5. Let A be an $n \times m$ matrix with $n > m$ Consider the matrix $B = A(A^tA)^{-1}A^t$
   (a) Show that $B^2 = B$.
   (b) Show that $BA = A$.

6. Suppose that $T : \mathbb{R}^4 \to \mathbb{R}^3$ is represented by the matrix

$$A = \begin{pmatrix} 1 & 2 & 1 & 1 \\ 2 & 1 & 0 & 2 \\ -1 & 4 & 3 & -1 \end{pmatrix}$$

   in the usual basis of $\mathbb{R}^3$ and $\mathbb{R}^4$.

(a) Find the null space or kernel of T.

(b) Find the vector space spanned by the columns.

(c) Check that the dimensions of the spaces in part (a), (b) satisfy the appropriate equality.